## Session T – Mathematics/ Applied and Computational Mathematics
(Alphabetical)

### Infinite Walks on the Primes of Integer Lattices
Ana S Balibanu
*Mentor: Prof. Dinakar Ramakrishnan*

In rings that exhibit unique factorization into primes, the question arises of whether it is possible, given a bounded step size, to “walk” out to infinity by stepping only on primes. We approach this problem in the ring of Eisenstein integers by finding an angle measure so that no infinite walk can exist within angular sectors of that measure. We then find collections of such sectors that approach the plane and do not admit infinite walks. We prove analogous results for the union of the sets of Gaussian and Eisenstein primes. We then inspect the problem from other perspectives, including that of percolation theory.

### Structural Optimization Using Sensitivity Analysis and a Level Set Method
Anton Bongio Karrman
*Mentor: Grégoire Allaire, Professor of Applied Mathematics at Ecole Polytechnique, France*

A common problem in mechanical structure design is to optimize the shape and topology of an elastic structure given certain boundary conditions; the optimal structure usually performs some desired function and is lightweight but strong. Shape optimization is usually quite straightforward unlike topology optimization, which requires advanced techniques because topology variation usually causes discontinuous changes in the physical properties of structures. A shape and topology optimization algorithm developed in recent years uses the classical shape derivative paired with the level set method in order to find the optimal shape and topology of elastic structures given certain loads and conditions. In this project, using finite element analysis along with numerical discretization schemes that solve the Hamilton-Jacobi level set equation, we implement this algorithm by applying it to several test cases and present the code in Scilab.

### Searching for Multigrades
Zarathustra Brady
*Mentor: Matthias Flach*

An (n,k) multigrade is defined to be a pair of sets of n numbers that have equal sums, sums of squares, and so on up to kth powers. The Prouhet-Tarry-Escott problem is to find integer multigrades with $n = k+1$ (called ideal multigrades). For $k \leq 7$, parametric solutions have been found to the multigrade problem. We attempt to find more parametric solutions by finding curves contained in the set of trivial solutions (i.e., both sets are the same), and deforming them out and into the nontrivial solutions.

### Noncommutative Geometry Techniques Applied to Quantum Gravity
Domenic Denicola
*Mentor: Matilde Marcolli*

Over the last thirty years, a new set of mathematical techniques, under the collective heading of noncommutative geometry, have shown great promise as a foundation for many aspects of contemporary physics, including the famously-successful Standard Model. It is thus natural to suppose that noncommutative geometry would be useful in our attempts to unify quantum field theory and general relativity into a theory of quantum gravity, one of the most prominent open problems in modern physics. We investigate in detail how to apply noncommutative techniques to problems of quantum gravity, finding that several modifications to the mathematical framework—such as the extension to connections with torsion—to be crucial. This project develops these modifications, and explores their consequences for applications to quantum gravity.

### Reflectarray Design with Rigorous Scattering Solvers and Integral Equations
Ryan Denlinger
*Mentor: Oscar Bruno*

A reflectarray is a microstrip array antenna whose patch sizes are varied across the face of the array so as to focus electromagnetic waves in a well-defined direction; for instance, a typical reflectarray design might focus light from a point-source (feed) into an outgoing plane wave, like a parabolic mirror. Although reflectarrays operate over very narrow bandwidths, they are cheap, flat and easily folded, making them useful for space-based applications. A reflectarray will generally have hundreds or thousands of radiating elements, and good design requires optimizing over the properties of each element, so efficient and accurate computational methods are crucial. A new code is developed for rigorously modeling reflectarrays in two dimensions. The code is then used to determine the numerical accuracy of a full-wave moment method commonly used to design reflectarrays.

**On Free Groups Generated by Two Parabolic Elements of SL(2, Q)**
Casey Jao
Mentors: *Danny Calegari and Matt Day*

An open question asks whether there exists rational x in (-2, 2) such that the group generated by the matrices (1, x; 0, 1) and (1, 0; x, 1) is free. A well-known dynamical argument shows that this group is free whenever |x| >= 2. Using topological and algebraic methods, we study the group when |x| < 2, where this argument breaks down.

**Numerical Approximations to Renormalization Group Equations for Standard Model with Neutrino Mixing**
Daniel Kolodrubetz
*Mentor: Matilde Marcolli*

Renormalization Group Equations govern the development of parameters of the standard model through a range of energy levels. It is impossible to solve these equations analytically, so approximations must be made. By using Mathematica, the running of several important parameters can been done in order see the development of a new model through energies from the electroweak scale to unification energy. Specifically, the new model involves a noncommutative geometry and gravity terms in its spectral analysis. Also, there are different boundary conditions that apply to the new model. In order to get an appropriate running, both the parameters and boundary conditions were met in the Mathematica program. The analysis of the development of these gravity terms can lead to cosmological implications, though this precise analysis will be left for further research.

**Stochastic Loewner Evolutions and the Ginibre-Girko Ensemble**
Ben McMillan
*Mentor: Nikolai Makarov*

We provide numerical evidence that the chordal level curves of the Ginibre-Girko ensemble potentials converge to the Stochastic Loewner Evolution of parameter 4. In particular, SLE theory predicts that the probability of a point on the disk being above a sample SLE(4) curve is a harmonic function on the disk. We show using Monte Carlo simulations that the distribution of this random variable for the family of level curves is remarkably close to the predictions of SLE(4). This could have implications for the potential level curves of more general random matrix ensembles.

**Generalized Iterated Function Systems**
Alexandra M Musat
*Mentors: Radu Miculescu and Danny Calegari*

The purpose of the project is to give a generalization of the notion of iterated function systems. (GIFS). Given a complete metric space $(X,d)$, we extend the notion of iterated function systems –IFS (i.e.: iterated function system- a metric space, together with a finite set of contractions), by considering contractions $(f_i)_{i=\overline{1,n}}$, $f_i : X^m \to X$. These generalizations keep some of the properties of IFS: starting with the finite family of contractions $(f_i)_{i=\overline{1,n}}$, $f_i : X^m \to X$, we can define a function $F_S : K^*(X)^m \to K^*(X)$ ($K^*(X)$ denotes the fractal space, consisting of the compact subsets of X, other than the empty set), such that $F_S(K_1,\dots,K_m) = \bigcup_{i=\overline{1,n}} F_{f_i}(K_1,\dots,K_m)$, where $F_f(K_1,\dots,K_m) = \{f(x_1,\dots,x_m) \mid x_i \in K_i, \forall i \in \{1,\dots,m\}\}$, $F_f : K^*(X)^m \to K^*(X)$ . After proving that this function is a contraction, it follows that $F_S$ has a unique fixed point- the attractor, the notion of attractor being thus extended from IFS to GIFS. We also aim to give examples of GIFS and find their attractors.

**Analogues of the Curve Complex**
Michael Smith
*Mentor: Danny Calegari*

The outer automorphism groups of free groups have been an important object of study in the field of geometric group theory. The program of study has followed in close analogy to the study of mapping class groups. One object that appears in the study of mapping class groups is the curve complex of a surface. This is a simplicial complex with a vertex for every homotopy class of essential simple closed curves in the surface, and an *n*-simplex on (*n*+1) vertices if the curves these vertices represent can be disjointly realized on the surface. There is no agreed-upon analogue of the curve complex in the study of free group automorphisms, although there are several candidates. I address some questions regarding the geometry of one of these candidates and its relationship to other complexes that $Out(F_n)$ act on.

**New Approaches to Computing the Spectra and Pseudospectra of Hermitian and Non-Hermitian Schrödinger Operators**

Christopher A. Wong
*Mentor: Anders C. Hansen*

In this paper we consider the question of how to compute spectra and pseudospectra of Hermitian and non-Hermitian Schrödinger operators. We avoid discretizations of the Laplacian, as these may give very deceiving results in the non-Hermitian case, and instead focus on a new theory that is based on using matrix elements from the infinite matrix induced by the Schrödinger operator and a suitable basis (typically a core). The numerical methods come with theorems guaranteeing convergence.